

# Matrices

## General rules and properties of matrices

- Order of any matrix: Row  $\times$  Column  
A  $2 \times 3$  matrix will have 2 rows and 3 columns:

$$\begin{array}{ccc} & 3 \text{ Columns} & \\ & \downarrow \downarrow \downarrow & \\ 2 \text{ rows} & \rightarrow \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} & \end{array}$$

- Row Matrix: Matrix with exactly one row
- Column Matrix: Matrix with exactly one column
- Matrices ( $A$  and  $B$ ) of the same order are
  1. Commutative:  $A + B = B + A$
  2. Associative:  $(A + B) + C = A + (B + C)$

## Addition/subtraction of matrices

Order of the matrix must be exactly the same.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a \pm 1 & b \pm 2 \\ c \pm 3 & d \pm 4 \end{pmatrix}$$

## Multiplication of matrix by a scalar/real number

The scalar multiplies into the entire matrix regardless of the order.

$$\begin{aligned} k \begin{pmatrix} a & b \\ c & d \end{pmatrix} &= \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix} \\ h \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} &= \begin{pmatrix} ha & hb \\ hc & hd \\ he & hf \end{pmatrix} \end{aligned}$$

## Multiplication of 2 matrices

- In order for 2 matrices to be able to multiply, the column of the 1st matrix must be equal to the row of the 2nd matrix
- The order of the resultant matrix would be the the (row of the 1st matrix)  $\times$  (column of the 2nd matrix)
- If the order allows for the matrix multiplication, the row of the 1st matrix will always multiply to the column of the 2nd matrix. Add them up to find the new product of the matrices

Examples:

A  $2 \times 3$  and a  $3 \times 1$  matrix multiplication will give rise to a  $2 \times 1$  matrix

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \times \begin{pmatrix} g \\ h \\ i \end{pmatrix} = \begin{pmatrix} ag + bh + ci \\ dg + eh + fi \end{pmatrix}$$

A  $3 \times 1$  and a  $1 \times 3$  matrix multiplication will give rise to a  $3 \times 3$  matrix

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d & e & f \end{pmatrix} = \begin{pmatrix} ad & ae & af \\ bd & be & bf \\ cd & ce & cf \end{pmatrix}$$

A  $2 \times 2$  and a  $2 \times 3$  matrix multiplication will give rise to a  $2 \times 3$  matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} e & f & g \\ h & i & j \end{pmatrix} = \begin{pmatrix} ae + bh & af + bi & ag + bj \\ ce + dh & cf + di & cg + dj \end{pmatrix}$$

## Tackling matrix problem sums

- When constructing the matrix, always make sure that the order of the matrices are strictly adhered to
- Check-back and ensure that the multiplication makes sense.

For example:

Total revenue of a theme park is calculated by

$= (\text{number of child tickets sold} \times \text{price of each child ticket}) + (\text{number of adult tickets sold} \times \text{price of each adult ticket})$

If your matrix multiplication shows otherwise it means the terms have been ordered incorrectly